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A PREDICTIVE ANALYTIC MODEL FOR SOLAR MODULATION

with Ilias Cholis, Dan Hooper (based on: 1511.01507, PRD)

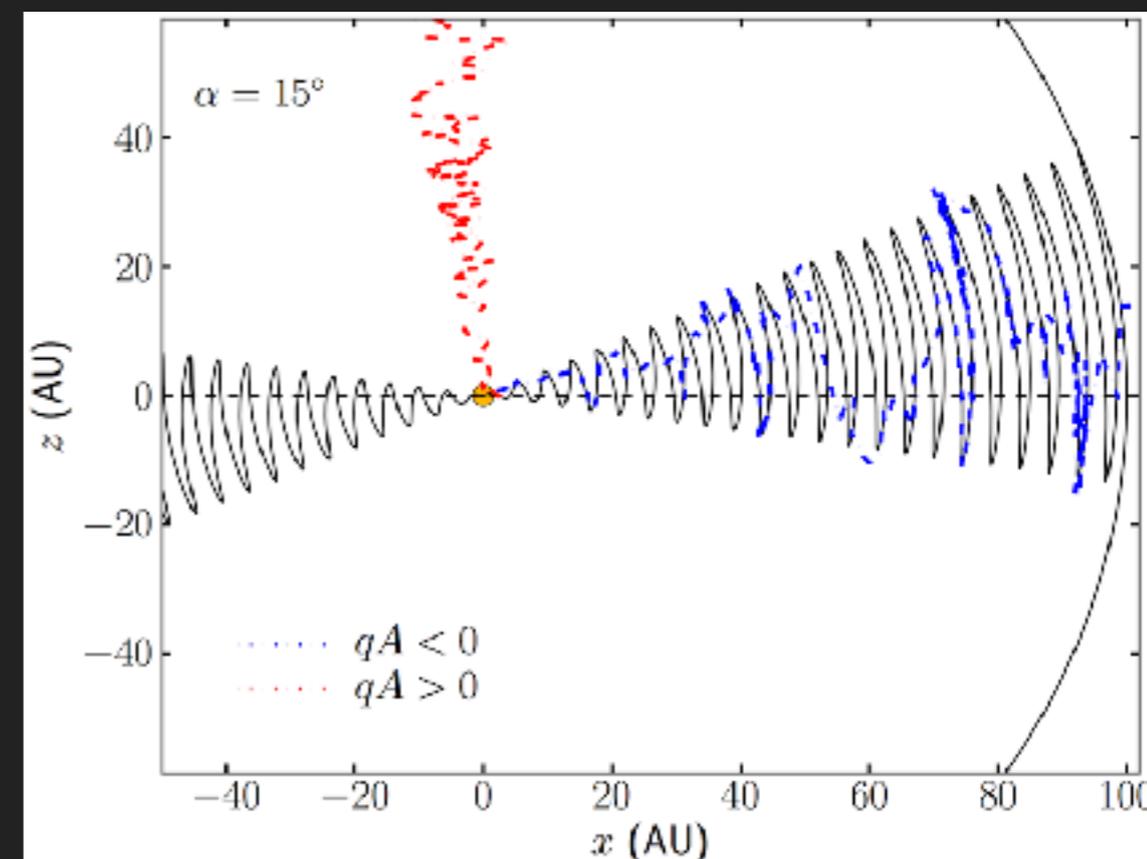
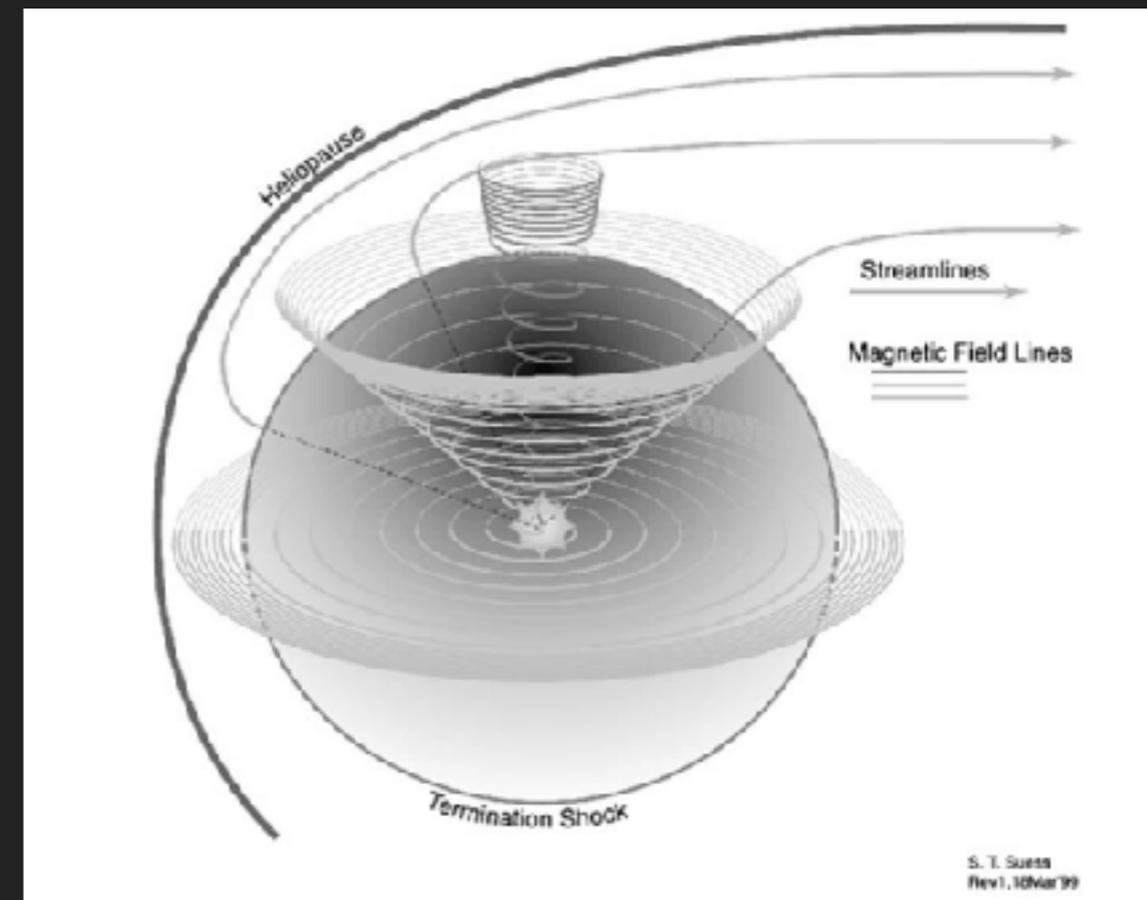
SOLAR ENERGETIC PARTICLES CONFERENCE

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THE PHYSICS OF SOLAR MODULATION

- ▶ As cosmic-rays approach Earth, they undergo energy losses in the heliospheric magnetic field.
- ▶ The magnetic field of the sun is modeled by a Parker spiral, with a heliospheric current sheet along the galactic plane.
- ▶ Particles with $qA > 0$ propagate easily along the poles, particles with $qA < 0$ must move through the heliospheric current sheet.



THE PHYSICS OF SOLAR MODULATION

CR phase space density

CR diffusion

CR production inside heliosphere

$$\frac{\partial f}{\partial t} = -(\vec{V} + \langle \vec{v}_D \rangle) \nabla f + \nabla(\hat{D} \nabla f) + \underbrace{\frac{1}{3}(\nabla \vec{V})}_{\text{Adiabatic Energy Losses}} \frac{\partial f}{\partial \ln p} + J_{\text{source}}$$

Solar wind velocity

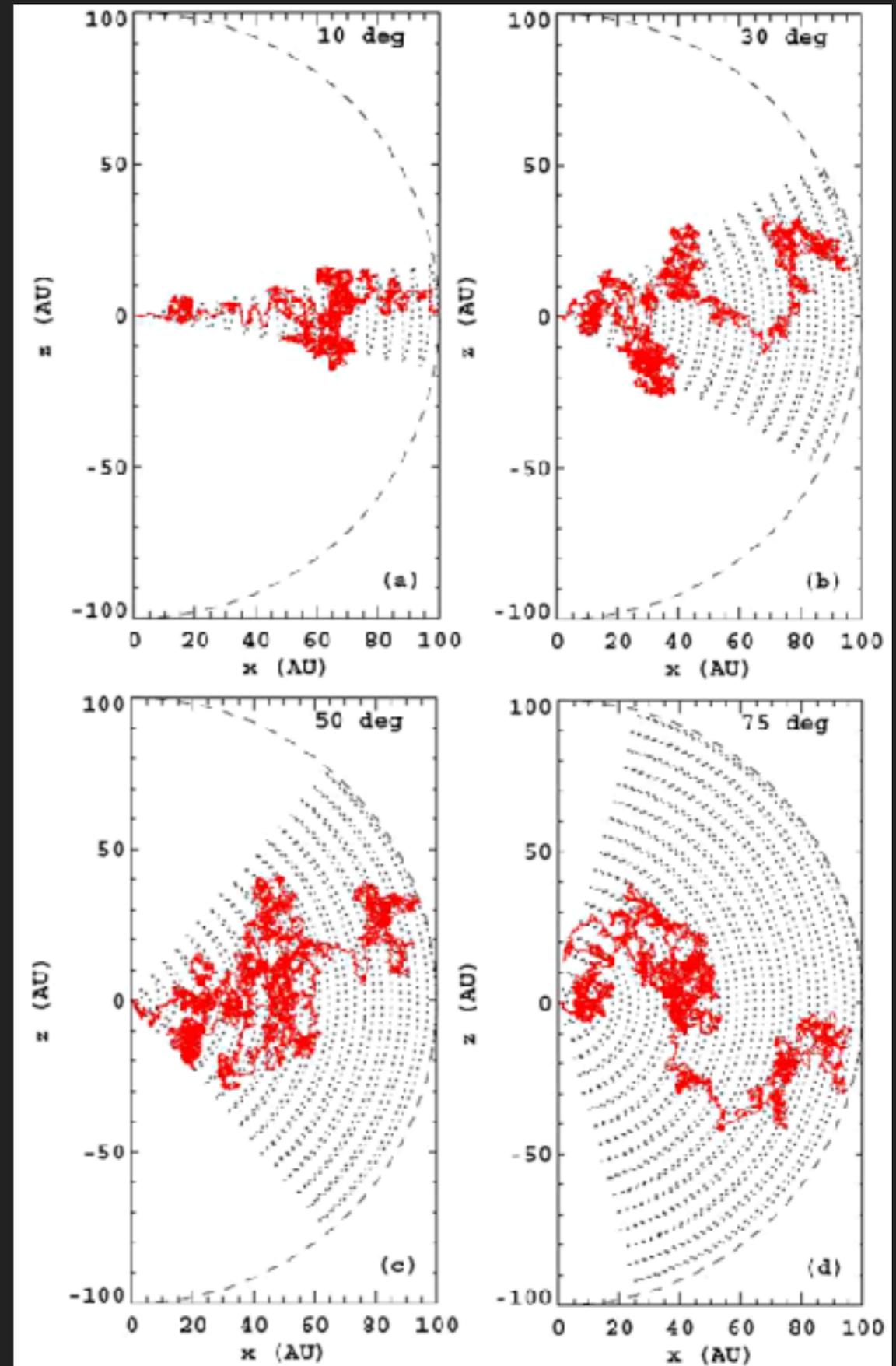
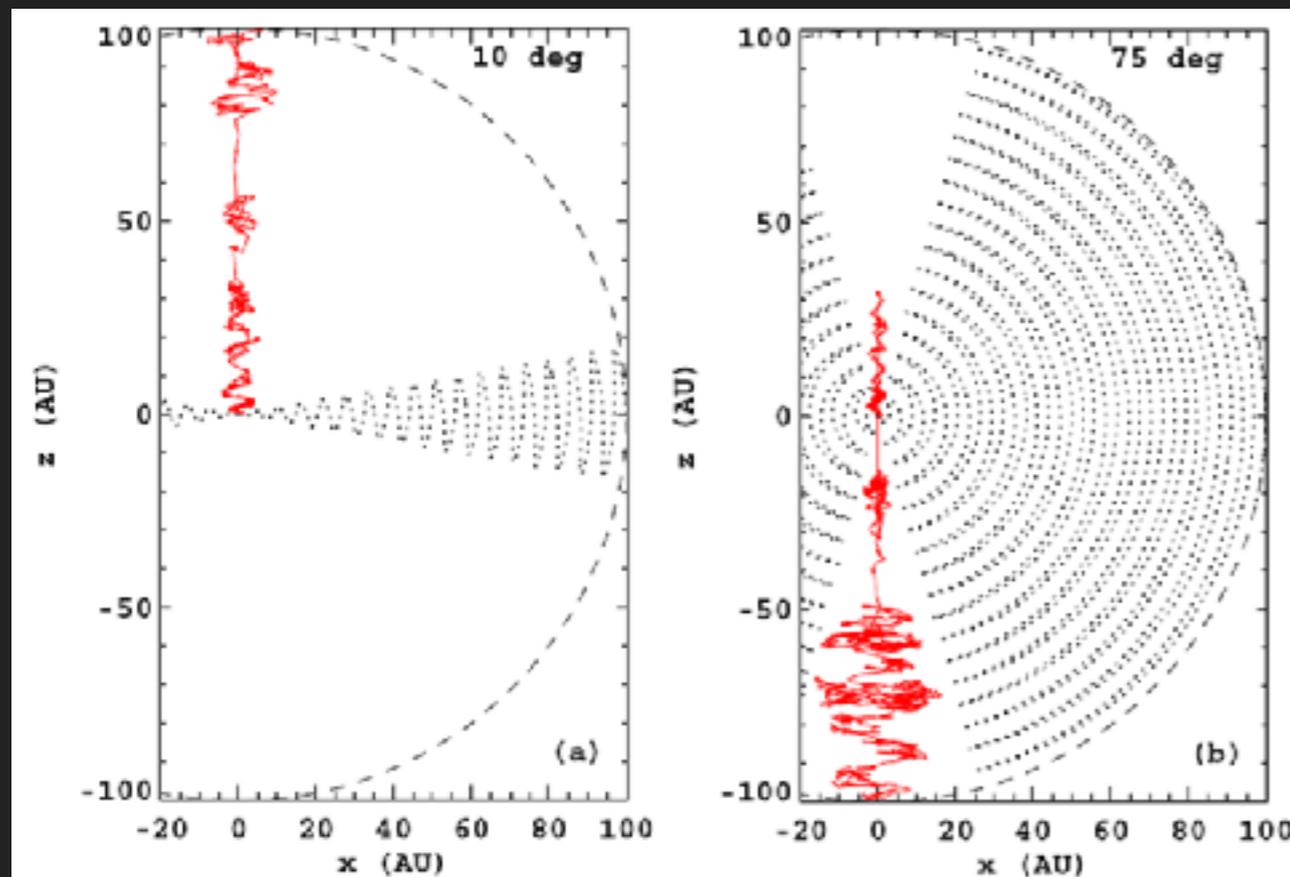
drift velocity

Adiabatic Energy Losses

- ▶ Cosmic-Rays move through the magnetic field through a combination of diffusion and drift, losing energy adiabatically along the way.

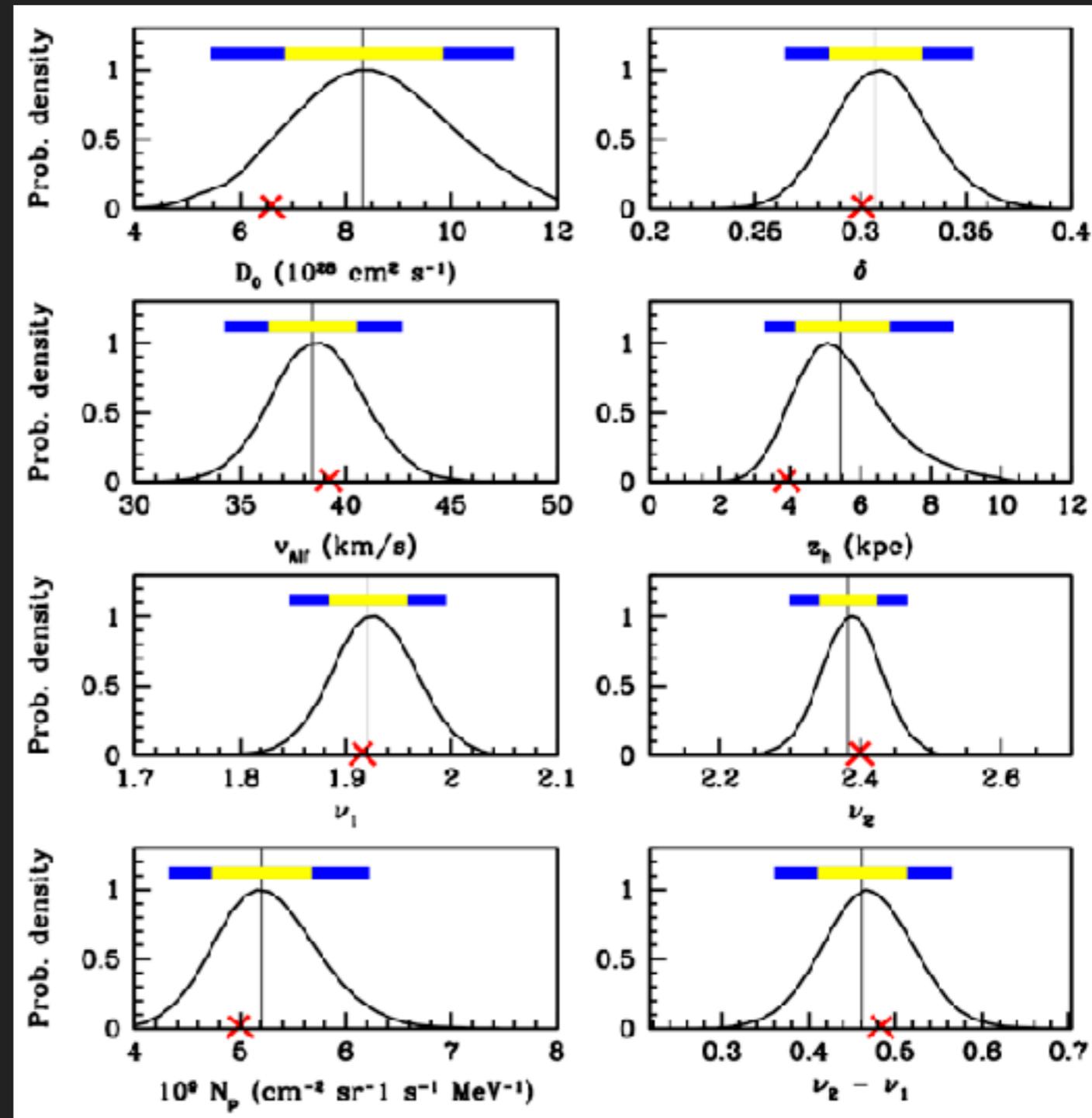
- ▶ Can determine the effect of solar modulation through direct calculation of the particle transport equation.
- ▶ These models are physically motivated, but computationally intensive.

Strauss et al. (2012)



ANOTHER GOAL: UNDERSTANDING THE INTERSTELLAR MEDIUM

- ▶ Cosmic-Ray production, propagation and energy losses in the interstellar medium are also extremely complex.
- ▶ Computational models have calculated the correlated systematics between effects such as diffusion, the diffusive halo height, Alfvén reacceleration etc.



Trotta et al. (2011, 1011.0037)

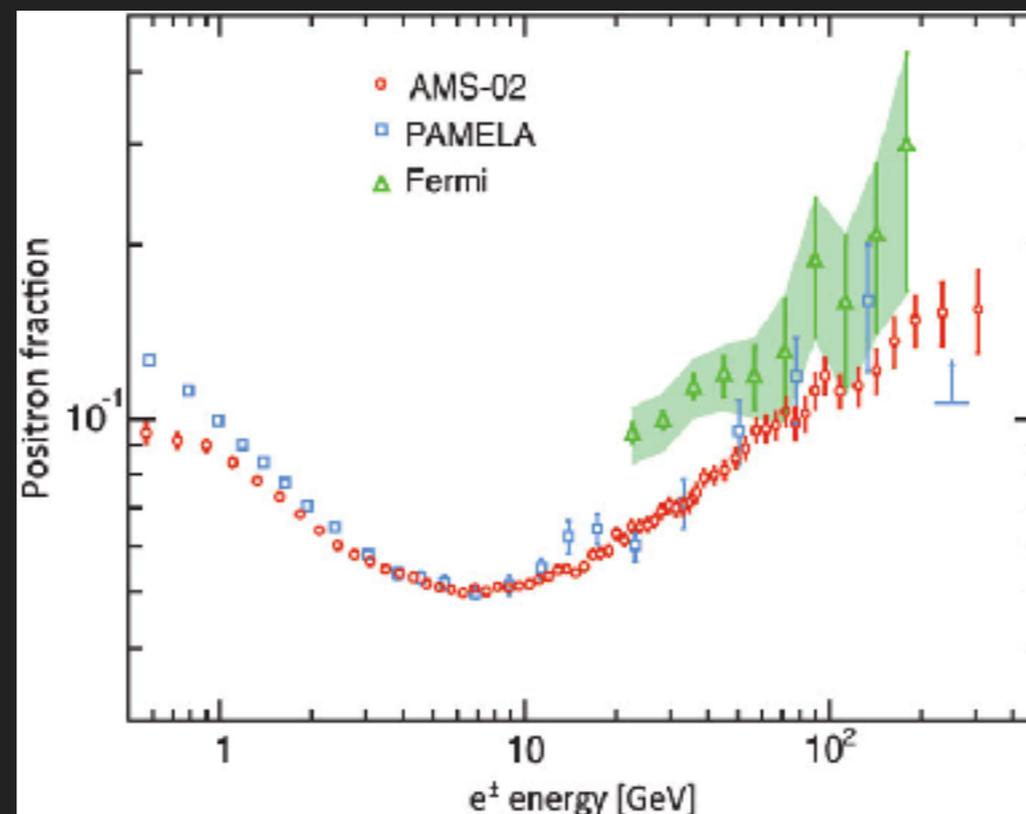
$$\frac{\partial \psi(r, p, t)}{\partial t} = q(r, p, t) + \vec{\nabla} \cdot (D_{xx} \vec{\nabla} \psi) + \frac{\partial}{\partial p} \left[p^2 D_{pp} \frac{\partial}{\partial p} \left(\frac{\psi}{p^2} \right) \right] + \frac{\partial}{\partial p} \left[\frac{p}{3} (\vec{\nabla} \cdot \vec{V}) \psi \right]$$

ANOTHER GOAL: UNDERSTANDING THE INTERSTELLAR MEDIUM

The Solution is Simple!

$$\frac{\partial \psi(r, p, t)}{\partial t} = q(r, p, t) + \vec{\nabla} \cdot (D_{xx} \vec{\nabla} \psi) + \frac{\partial}{\partial p} \left[p^2 D_{pp} \frac{\partial}{\partial p} \left(\frac{\psi}{p^2} \right) \right] + \frac{\partial}{\partial p} \left[\frac{p}{3} (\vec{\nabla} \cdot \vec{V}) \psi \right]$$

$$\frac{\partial f}{\partial t} = -(\vec{V} + \langle \vec{v}_D \rangle) \nabla f + \nabla (\hat{D} \nabla f) + \frac{1}{3} (\nabla \vec{V}) \frac{\partial f}{\partial \ln p} + J_{\text{source}}$$



PLAN B: SIMPLIFY THE HELL OUT OF EVERYTHING

$$\frac{\partial f}{\partial t} = -(\vec{V} + \langle \vec{v}_D \rangle) \nabla f + \nabla (\hat{D} \nabla f) + \frac{1}{3} (\nabla \vec{V}) \frac{\partial f}{\partial \ln p} + J_{\text{source}}$$

Steady State: $\frac{\partial f}{\partial t} = 0$

No Sources: $J_{\text{source}} = 0$

Spherical Symmetry: $\vec{V} \rightarrow V$
 $f \rightarrow f(r)$

Isotropic Diffusion: $\hat{D} \rightarrow D$

No Drift: $\langle v_D \rangle = 0$

then the model simplifies to diffusion with energy loss (i.e. climbing a potential). We get:

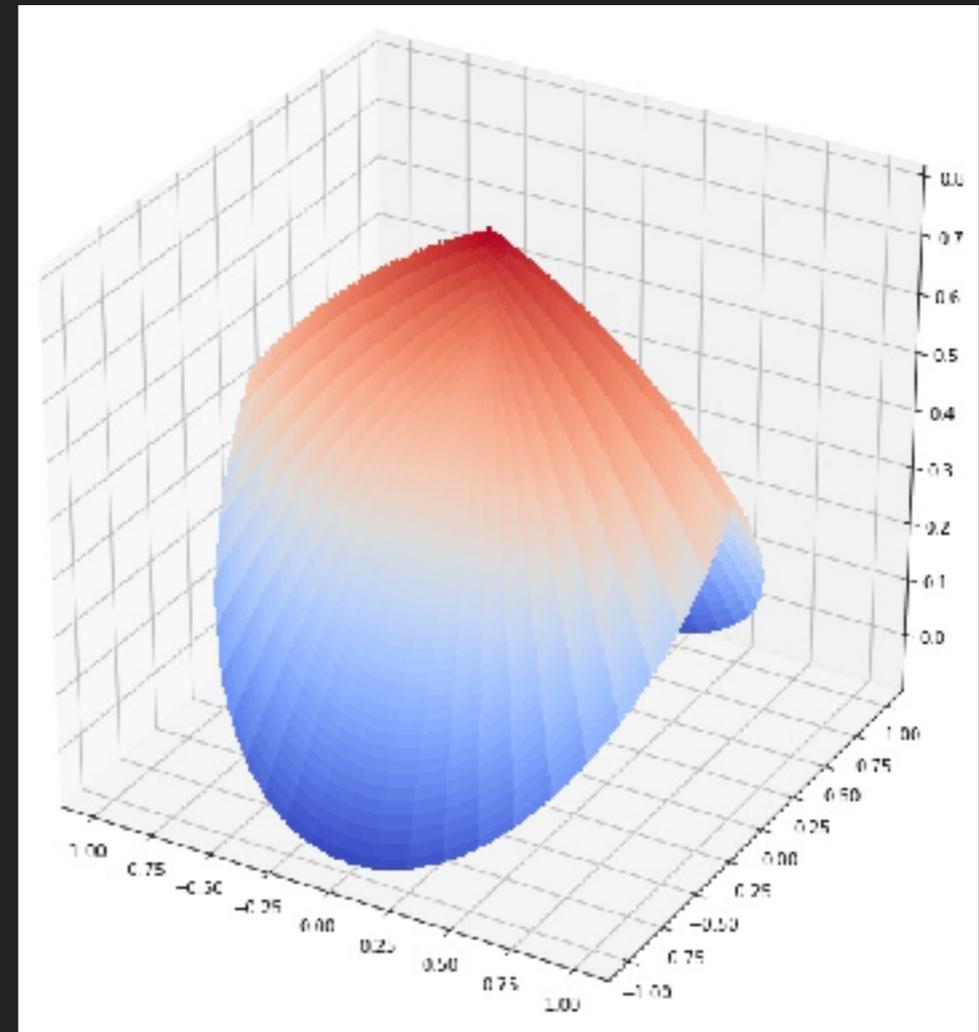
$$\frac{\partial f}{\partial r} + \frac{V}{3D} \frac{\partial f}{\partial \ln(p)} = 0$$

$$\frac{dN^\oplus}{dE_{kin}}(E_{kin}) = \frac{(E_{kin} + m)^2 - m^2}{(E_{kin} + m + |Z| e\Phi)^2 - m^2} \frac{dN^{\text{ISM}}}{dE_{kin}}(E_{kin} + |Z| e\Phi)$$

FORCE FIELD APPROXIMATIONS OF SOLAR MODULATION

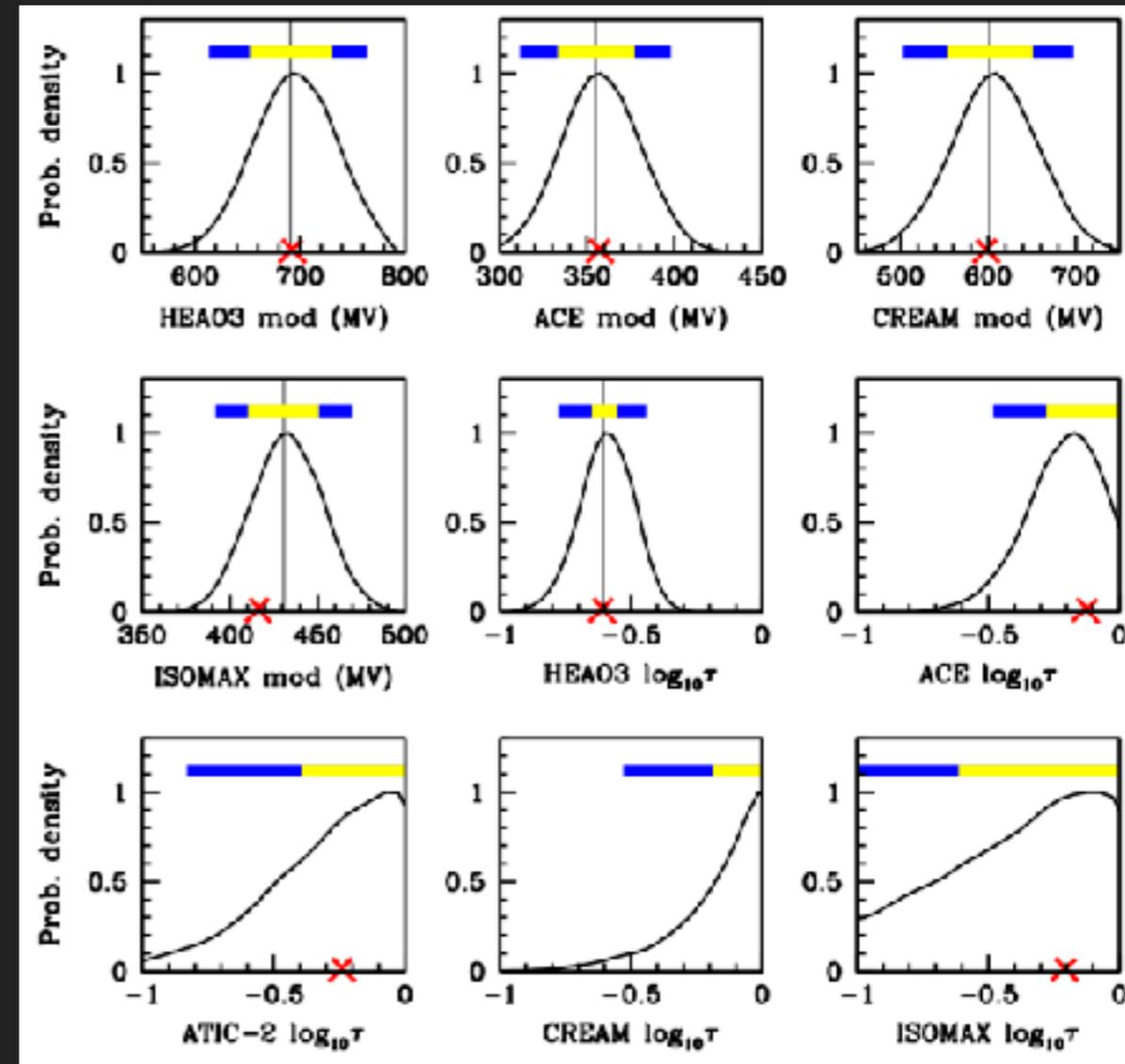
$$\frac{dN^{\oplus}}{dE_{kin}}(E_{kin}) = \frac{(E_{kin} + m)^2 - m^2}{(E_{kin} + m + |Z| e\Phi)^2 - m^2} \frac{dN^{ISM}}{dE_{kin}}(E_{kin} + |Z| e\Phi)$$

- ▶ Solar Modulation can also be treated as a simple potential, which particles must climb before reaching Earth.
- ▶ Two Effects:
 - ▶ 1.) The flux of particles is decreased
 - ▶ 2.) Particles that do climb the potential lose energy, implying that an Earth bound experiments experiment probes a higher energy ISM flux.
 - ▶ 3.) The model can include a charge-dependent modulation potential



UNDERSTANDING THE ISM IN THE FORCE-FIELD APPROXIMATION

- ▶ The force-field approximation allows for a fast analysis appropriate for scans of the ISM propagation parameter space
- ▶ But this adds uncertainty, since the modulation parameters must be fit for each observation.



Trotta et al. (2011, 1011.0037)

Quantity	Best fit value	Posterior mean and standard deviation	Posterior 95% range
DIFFUSION MODEL PARAMETERS Θ			
$D_0 (10^{28} \text{ cm}^2 \text{ s}^{-1})$	6.59	8.32 ± 1.46	[5.45, 11.20]
δ	0.30	0.31 ± 0.02	[0.26, 0.35]
$v_{\text{Alf}} (\text{km s}^{-1})$	39.2	38.4 ± 2.1	[34.2, 42.7]
$z_h (\text{kpc})$	3.9	5.4 ± 1.4	[3.2, 8.6]
ν_1	1.91	1.92 ± 0.04	[1.84, 2.00]
ν_2	2.40	2.38 ± 0.04	[2.29, 2.47]
$N_p (10^{-9} \text{ cm}^2 \text{ sr}^{-1} \text{ s}^{-1} \text{ MeV}^{-1})$	5.00	5.20 ± 0.48	[4.32, 6.23]
EXPERIMENTAL NUISANCE PARAMETERS			
Modulation parameters ϕ (MV)			
HEAO-3	693	690 ± 38	[613, 763]
ACE	357	354 ± 22	[311, 398]
CREAM	598	602 ± 49	[503, 697]
ISOMAX	416	430 ± 20	[391, 470]
ATIC-2	0 (fixed)	N/A	N/A
Variance rescaling parameters τ			
HEAO-3	-0.60	-0.60 ± 0.10	[-0.82, -0.41]
ACE	-0.12	N/A	> -0.49 (1-tail)
CREAM	0.00	N/A	> -0.53 (1-tail)
ISOMAX	-0.21	N/A	> -1.21 (1-tail)
ATIC-2	-0.24	N/A	> -0.84 (1-tail)

- ▶ These uncertainties are degenerate with our understanding of interstellar cosmic-ray propagation!

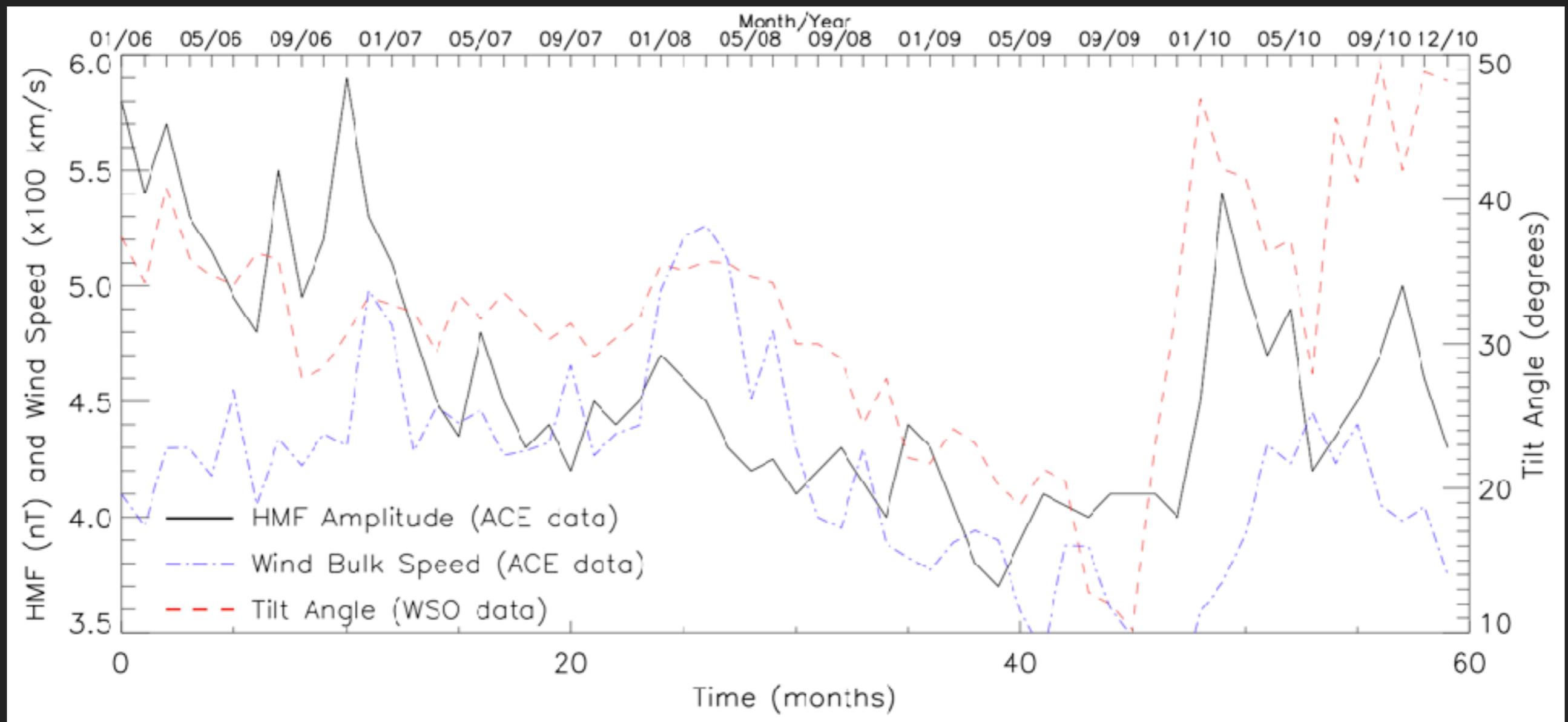
THE GOAL

- ▶ **Produce a model of solar modulation that:**
 - ▶ **1.) Takes into account some physical insights of the solar modulation of cosmic-rays.**
 - ▶ **2.) Provides accurate fits to the cosmic-ray data with few degrees of freedom.**
 - ▶ **3.) Can be computed in less than a second.**

▶ **Three Observations:**

- ▶ **1.) Solar Modulation Effects are time-dependent, interstellar medium effects are roughly time-independent.**
- ▶ **2.) Solar Modulation Effects are correlated to observed solar system properties, interstellar medium uncertainties are not.**
- ▶ **3.) Voyager data provides observations that are negligibly affected by solar modulation.**

BREAKING THE DEGENERACY: SOLAR OBSERVABLES



- ▶ In this study, we utilize two solar observables:
 - ▶ Amplitude of the heliospheric magnetic field at Earth (ACE)
 - ▶ Tilt Angle of the heliospheric current sheet (WSO)

BREAKING THE DEGENERACY: PHYSICAL INTUITION

- ▶ We start with the diffusion equation, and consider particle propagation along and perpendicular to the heliospheric current sheet separately. We assume J_{source} is negligible at these energies.

$$\frac{\partial f}{\partial t} = -(\vec{V} + \langle \vec{v}_D \rangle) \nabla f + \nabla (\hat{D} \nabla f) + \frac{1}{3} (\nabla \vec{V}) \frac{\partial f}{\partial \ln p} + J_{\text{source}}$$

- ▶ In the case of propagation at high heliolatitudes, drift is negligible, and propagation becomes proportional to the adiabatic energy loss rate and the cosmic-ray diffusion efficiency.

$$\Phi(R, t) = \phi_0 g(|B_{\text{tot}}(t)|)$$

BREAKING THE DEGENERACY: PHYSICAL INTUITION

$$\frac{\partial f}{\partial t} = -(\vec{V} + \langle \vec{v}_D \rangle) \nabla f + \nabla(\hat{D} \nabla f) + \frac{1}{3}(\nabla \vec{V}) \frac{\partial f}{\partial \ln p} + J_{\text{source}}$$

- ▶ In the case of propagation along the heliospheric current sheet, drift dominates for typical values of the heliospheric tilt angle.

$$\lambda_d = r_{\text{Larmor}} \frac{(R/R_0)^2}{1 + (R/R_0)^2}$$

- ▶ This allows drift at the Larmor radius at high rigidity, but significantly inhibits drift at low rigidities.
- ▶ Since the Larmor radius is inversely proportional to B, the propagation time (and total adiabatic energy loss) can be expressed as:

$$\tau_D \propto \frac{1}{|\langle \vec{v}_D \rangle|} \propto B(t) \frac{1 + (R/R_0)^2}{\beta (R/R_0)^3}$$

BREAKING THE DEGENERACY

- ▶ This motivates an additional term with a form:

$$\Phi(R, t) = g(|B_{\text{tot}}(t)|) f(\alpha(t)) \left(\frac{1 + (R/R_0)^2}{\beta(R/R_0)^3} \right)$$

- ▶ And a total potential:

$$\Phi(R, t) = \phi_0 g(|B_{\text{tot}}(t)|) + \phi_1 H(-qA) g(|B_{\text{tot}}(t)|) f(\alpha(t)) \left(\frac{1 + (R/R_0)^2}{\beta(R/R_0)^3} \right)$$

- ▶ Two More Assumptions:

- ▶ We assume that the function g is identical in each term, and noting the B^{-1} dependence of the Larmor radius, assume that the potential is proportional to B . We can fit the data with power-laws between 0 – 1.
- ▶ We fit $f(\alpha(t)) = \alpha^4$, based on results from the PAMELA and BESS data. We note simulations prefer a much smaller dependence.

THE SOLAR MODULATION POTENTIAL MODEL

$$\Phi(R, t) = \phi_0 \left(\frac{|B_{\text{tot}}(t)|}{4 \text{ nT}} \right) + \phi_1 H(-qA(t)) \left(\frac{|B_{\text{tot}}(t)|}{4 \text{ nT}} \right) \left(\frac{1 + (R/R_0)^2}{\beta(R/R_0)^3} \right) \left(\frac{\alpha(t)}{\pi/2} \right)^4$$

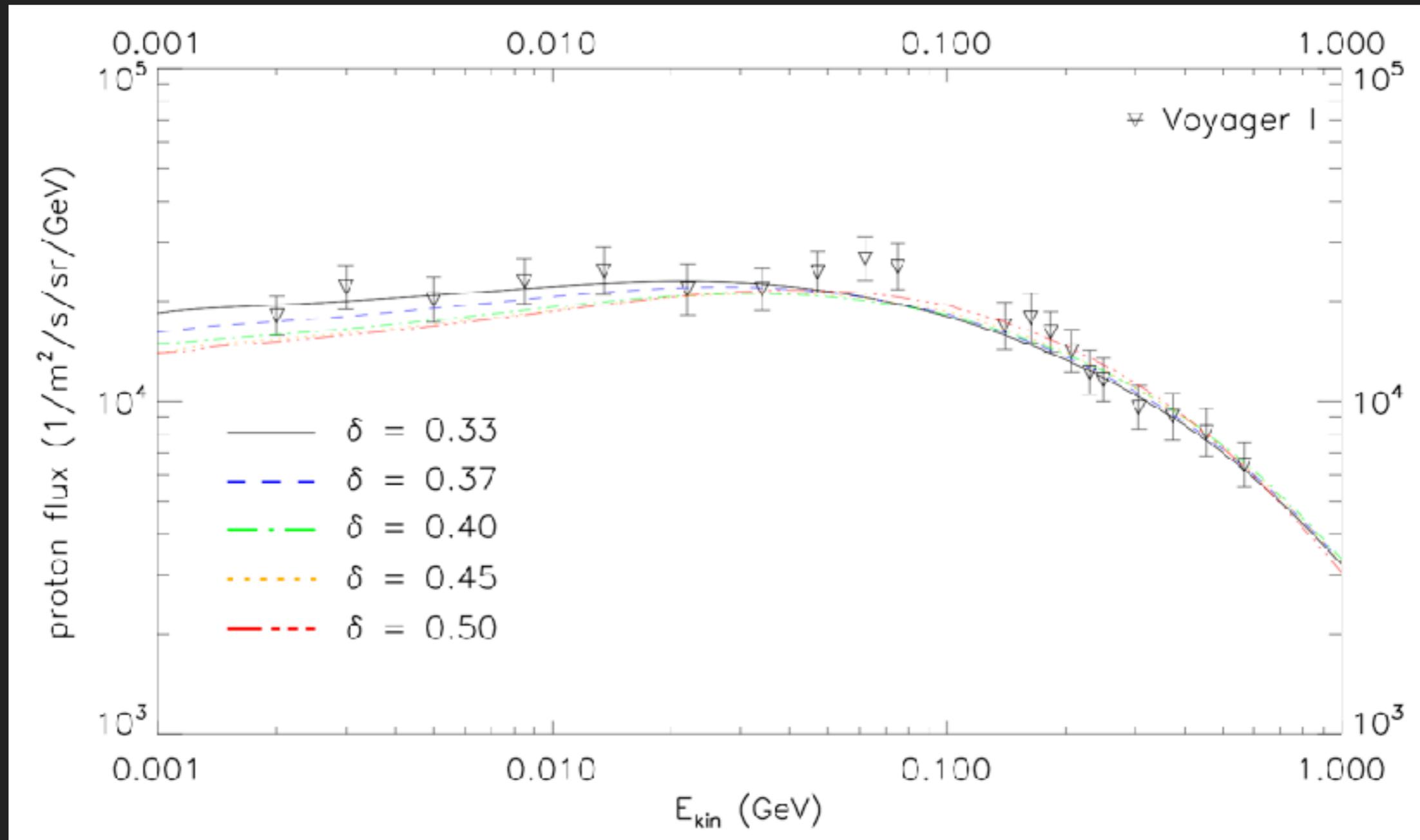
- ▶ **Benefits:**

- ▶ **Functional form of the potential is established.**
- ▶ **Can compare different datasets based on known solar observables.**

- ▶ **Drawbacks:**

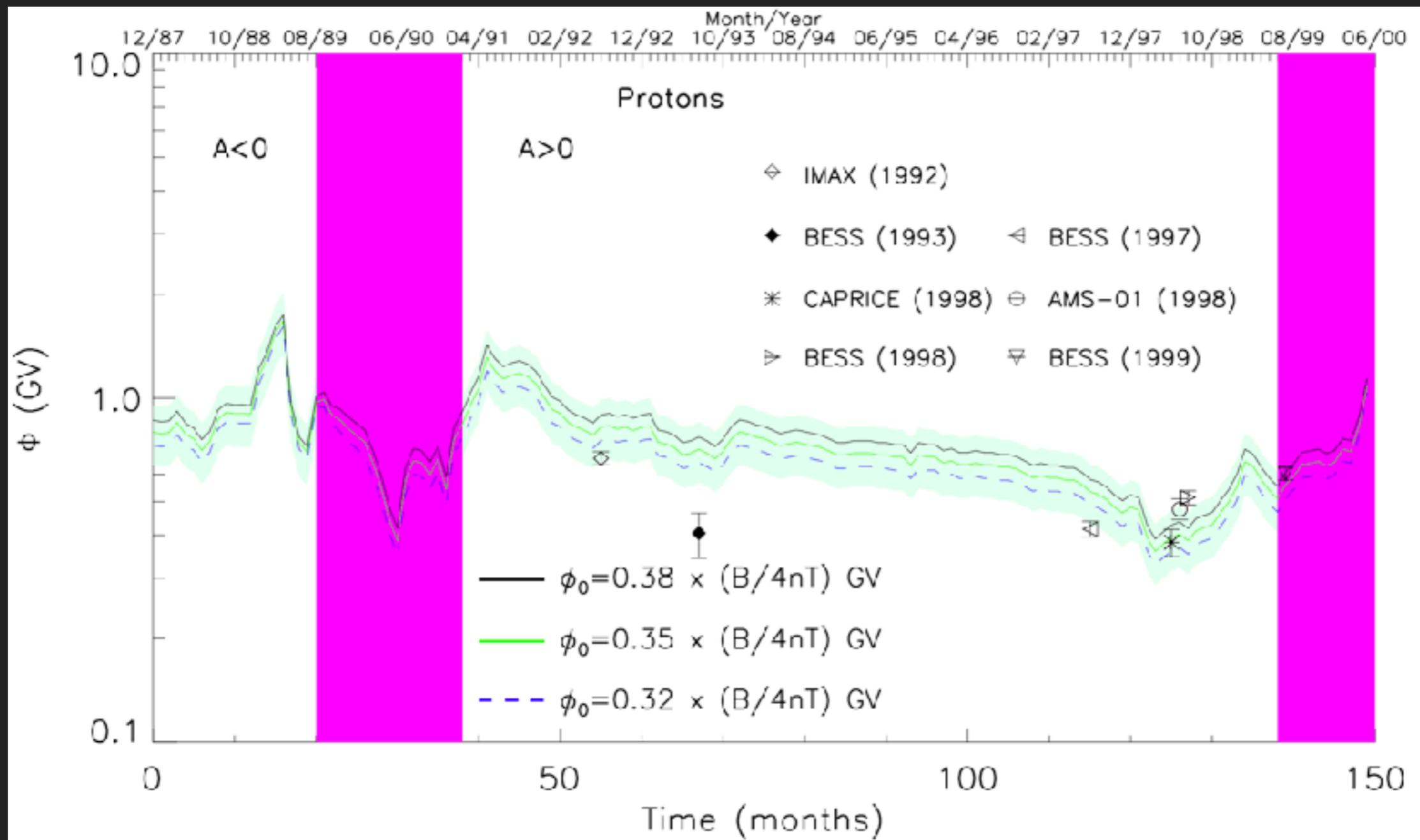
- ▶ **Still two unknown parameters, which must be fit in the analysis.**

CONSTRAINING THE FREE PARAMETERS WITH VOYAGER



- ▶ Can use Voyager data to break this degeneracy, by evaluating the cosmic-ray proton spectrum in a region without significant solar modulation.

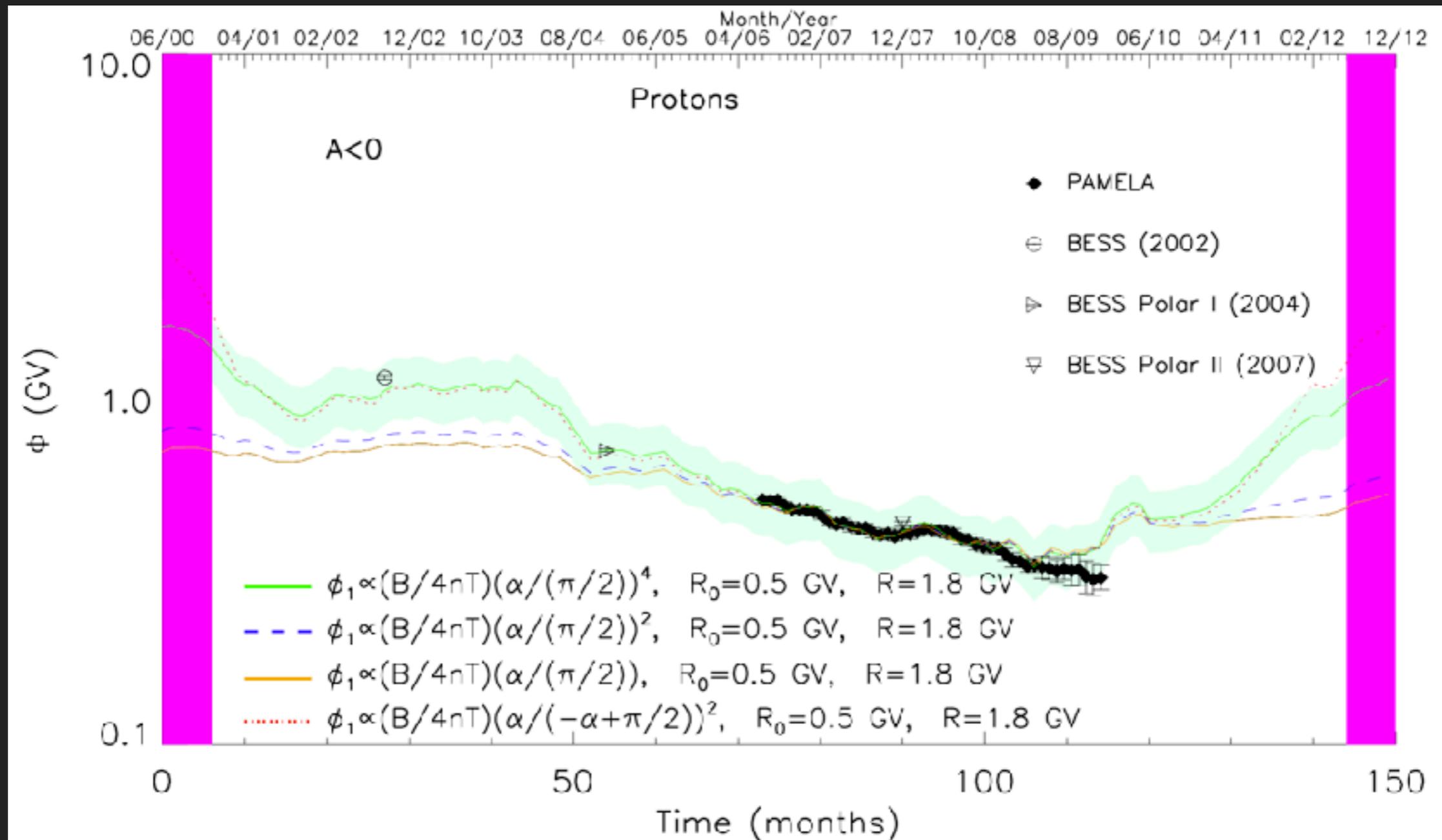
FITTING THE DATA: PROTONS WITH A>0



- ▶ We first fit the data in the simpler $qA > 0$ case, finding:

$$0.32 \text{ GV} < \phi_0 < 0.38 \text{ GV}$$

FITTING THE DATA: PROTONS WITH A<0



- ▶ Given this result, we then fit the functional dependence on the tilt of the heliospheric current sheet using PAMELA data, finding:

$$0.00 \text{ GV} < \phi_1 < 16.0 \text{ GV}$$

note: $(2\alpha/\pi)^4$

THE STORY OF THE TALK SO FAR

- ▶ From Theoretical Insights, we have developed a time- and charge-dependent form for the solar modulation potential which looks like:

$$\Phi(R, t) = \phi_0 \left(\frac{|B_{\text{tot}}(t)|}{4 \text{ nT}} \right) + \phi_1 H(-qA(t)) \left(\frac{|B_{\text{tot}}(t)|}{4 \text{ nT}} \right) \left(\frac{1 + (R/R_0)^2}{\beta(R/R_0)^3} \right) \left(\frac{\alpha(t)}{\pi/2} \right)^4$$

- ▶ By fitting to the observed, time-dependent proton flux, and utilizing observations from PAMELA, we have constrained the free-parameters in this fit to be:

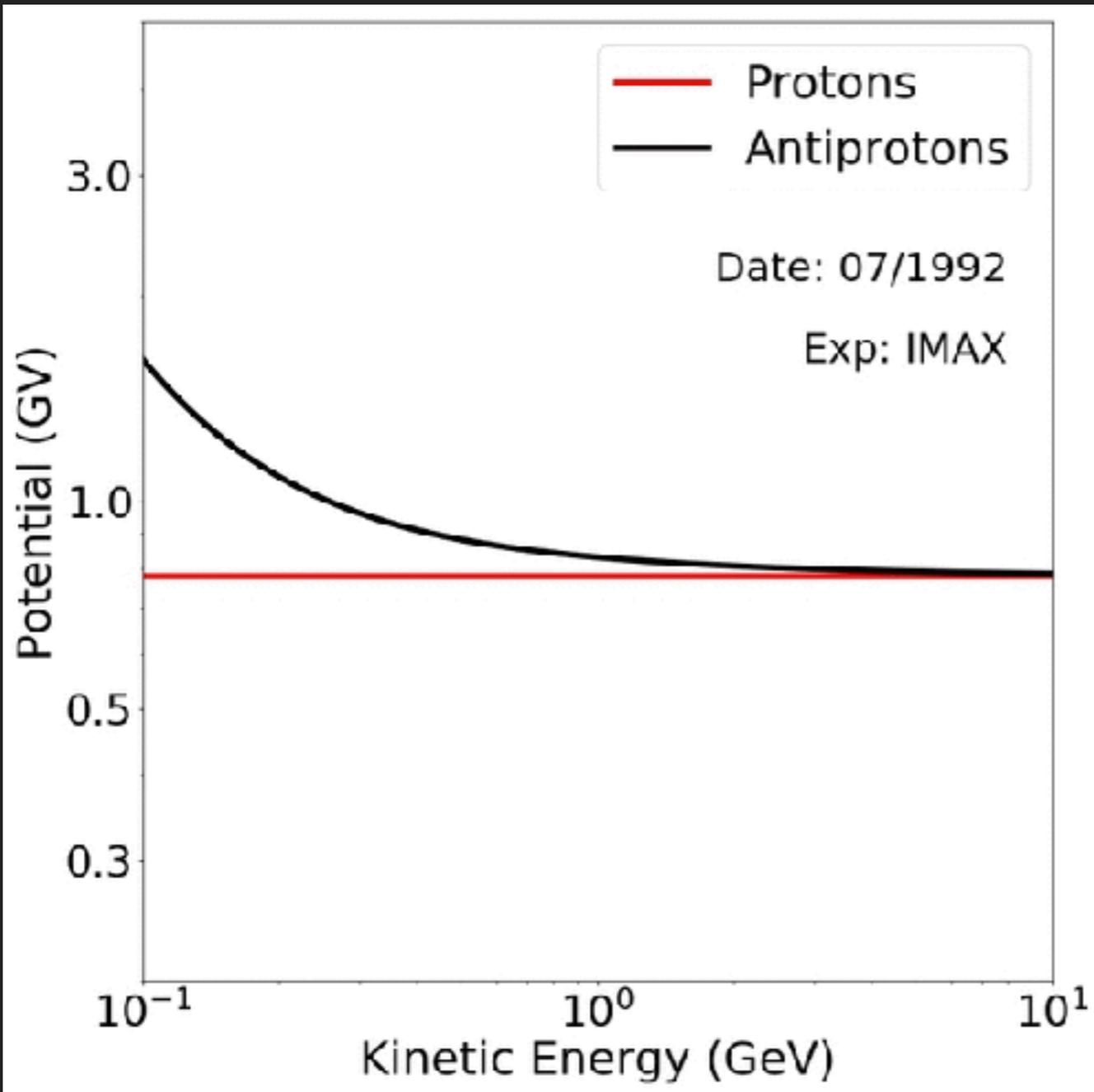
$$\phi_0 = 0.35 \text{ GV} \quad \phi_1 = 3.9 \text{ GV}$$

$$0.32 \text{ GV} < \phi_0 < 0.38 \text{ GV} \quad 0.00 \text{ GV} < \phi_1 < 16.0 \text{ GV}$$

- ▶ Which allows us to calculate the effect of solar modulation to be:

$$\frac{dN^\oplus}{dE_{\text{kin}}}(E_{\text{kin}}) = \frac{(E_{\text{kin}} + m)^2 - m^2}{(E_{\text{kin}} + m + |Z| e\Phi)^2 - m^2} \frac{dN^{\text{ISM}}}{dE_{\text{kin}}}(E_{\text{kin}} + |Z| e\Phi)$$

THE TIME DEPENDENCE IN THE MODULATION POTENTIAL



- ▶ This provides analytic solutions for the solar modulation potential as a function of time.
 - ▶ The computational time is similar to force-field approximation.
 - ▶ Given data of solar observables, the model is predictive for upcoming data from AMS-02.
-
- ▶ We use a model where the helicity changes continuously during periods where the helicity flips, though this modeling is uncertain.

THE TIME DEPENDENCE IN THE MODULATION POTENTIAL

Era	Exper.	$ B_{\text{tot}} $ (nT)	α (degrees)	$\Phi_{R=1GV}^{(q>0)}$	$\Phi_{R=2GV}^{(q>0)}$	$\Phi_{R=3GV}^{(q>0)}$	$\Phi_{R=1GV}^{(q<0)}$	$\Phi_{R=2GV}^{(q<0)}$	$\Phi_{R=3GV}^{(q<0)}$
07/92	IMAX	8.9	32.1	0.78	0.78	0.78	0.90 (0.89)	0.82 (0.82)	0.80 (0.80)
07/93	BESS	7.9	35.4	0.69	0.69	0.69	0.85 (0.80)	0.75 (0.73)	0.72 (0.71)
07/97	BESS	6.4	22.6	0.56	0.56	0.56	0.58 (0.62)	0.57 (0.58)	0.56 (0.57)
05/98	CAPRICE	4.3	46.3	0.38	0.38	0.38	0.63 (0.45)	0.46 (0.40)	0.43 (0.39)
06/98	<i>AMS-01</i>	4.5	45.2	0.39	0.39	0.39	0.63 (0.47)	0.48 (0.42)	0.44 (0.41)
07/98	BESS	4.6	46.6	0.40	0.40	0.40	0.68 (0.49)	0.50 (0.43)	0.46 (0.42)
07/99	BESS	5.8	73.9	0.51	0.51	0.51	2.71 (0.67)	1.26 (0.56)	0.97 (0.54)
08/02	BESS	7.6	55.1	1.54 (0.83)	0.96 (0.72)	0.85 (0.70)	0.66	0.66	0.66
12/04	BESS Polar I	6.4	46.5	0.95 (0.68)	0.69 (0.60)	0.64 (0.59)	0.56	0.56	0.56
07-12/06	<i>PAMELA</i>	5.2	34.2	0.54 (0.52)	0.48 (0.48)	0.47 (0.47)	0.45	0.45	0.45
01-06/07	<i>PAMELA</i>	4.9	32.1	0.49 (0.49)	0.45 (0.45)	0.44 (0.44)	0.43	0.43	0.43
07-12/07	<i>PAMELA</i>	4.4	31.1	0.44 (0.44)	0.40 (0.40)	0.40 (0.40)	0.39	0.39	0.39
12/07	BESS Polar II	4.5	32.5	0.45 (0.44)	0.41 (0.41)	0.40 (0.40)	0.39	0.39	0.39
01-06/08	<i>PAMELA</i>	4.5	34.7	0.47 (0.45)	0.42 (0.41)	0.41 (0.41)	0.39	0.39	0.39
07-12/08	<i>PAMELA</i>	4.2	28.8	0.40 (0.41)	0.38 (0.38)	0.37 (0.38)	0.37	0.37	0.37
01-06/09	<i>PAMELA</i>	4.0	21.5	0.36 (0.38)	0.36 (0.36)	0.35 (0.36)	0.35	0.35	0.35
07-12/09	<i>PAMELA</i>	4.1	18.7	0.36 (0.39)	0.36 (0.37)	0.36 (0.36)	0.36	0.36	0.36
01-06/10	<i>PAMELA</i>	4.7	39.7	0.56 (0.48)	0.46 (0.44)	0.44 (0.43)	0.41	0.41	0.41
07-12/10	<i>PAMELA</i>	4.6	39.9	0.55 (0.47)	0.45 (0.43)	0.43 (0.42)	0.40	0.40	0.40
01-06/11	<i>PAMELA</i>	4.7	48.3	0.73 (0.50)	0.52 (0.44)	0.48 (0.43)	0.41	0.41	0.41
07-12/11	<i>AMS-02 / PAMELA</i>	4.7	60.5	1.21 (0.52)	0.69 (0.45)	0.58 (0.43)	0.41	0.41	0.41
01-06/12	<i>AMS-02 / PAMELA</i>	4.8	67.2	1.66 (0.54)	0.85 (0.46)	0.68 (0.45)	0.42	0.42	0.42
01-06/14	<i>AMS-02</i>	5.3	67.3	0.46	0.46	0.46	1.83 (0.60)	0.92 (0.51)	0.75 (0.49)
07-12/14	<i>AMS-02</i>	5.6	62.0	0.49	0.49	0.49	1.54 (0.62)	0.85 (0.54)	0.71 (0.52)
01-06/15	<i>AMS-02</i>	6.6	56.6	0.58	0.58	0.58	1.44 (0.72)	0.87 (0.63)	0.76 (0.61)
07-12/15	<i>AMS-02</i>	7.0	51.5	0.61	0.61	0.61	1.24 (0.75)	0.83 (0.66)	0.74 (0.64)
01-06/16	<i>AMS-02</i>	6.7	48.8	0.59	0.59	0.59	1.07 (0.71)	0.75 (0.63)	0.69 (0.61)

THE GOAL: UNDERSTANDING THE INTERSTELLAR MEDIUM

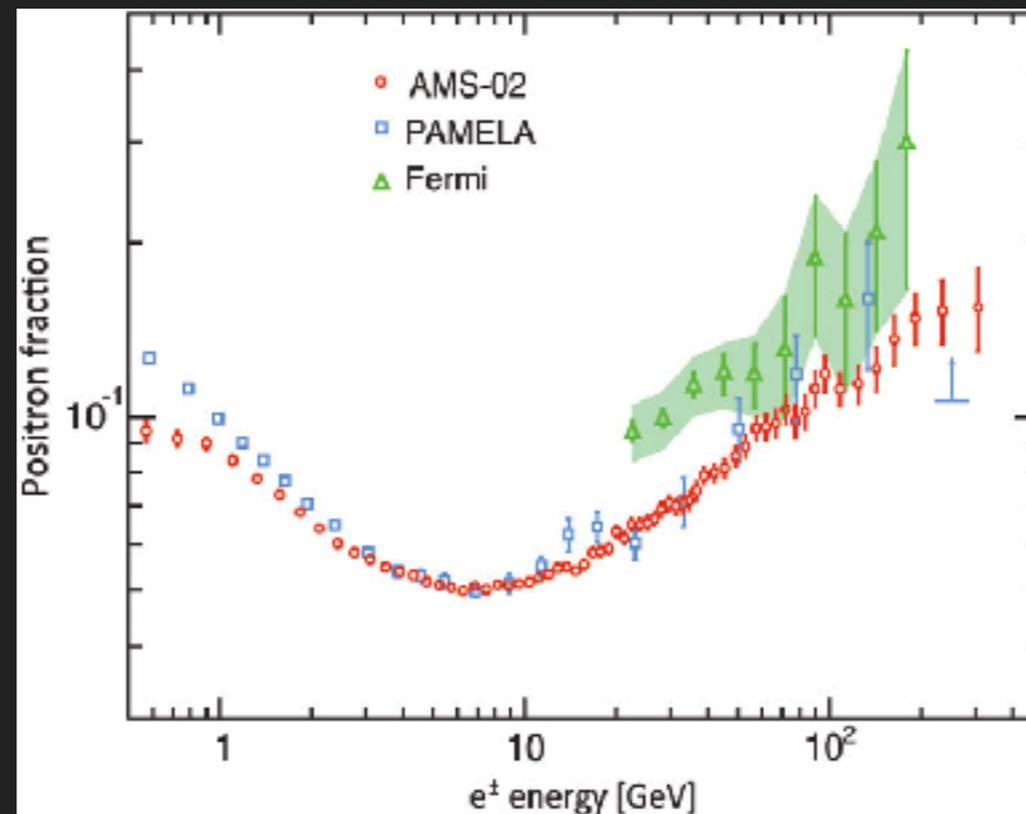
The Solution is Simple!

$$\frac{\partial \psi(r, p, t)}{\partial t} = q(r, p, t) + \vec{\nabla} \cdot (D_{xx} \vec{\nabla} \psi) + \frac{\partial}{\partial p} \left[p^2 D_{pp} \frac{\partial}{\partial p} \left(\frac{\psi}{p^2} \right) \right] + \frac{\partial}{\partial p} \left[\frac{p}{3} (\vec{\nabla} \cdot \vec{V}) \psi \right]$$

$$\Phi(R, t) = \phi_0 \left(\frac{|B_{\text{tot}}(t)|}{4 \text{ nT}} \right) + \phi_1 H(-qA(t)) \left(\frac{|B_{\text{tot}}(t)|}{4 \text{ nT}} \right) \left(\frac{1 + (R/R_0)^2}{\beta(R/R_0)^3} \right) \left(\frac{\alpha(t)}{\pi/2} \right)^4$$

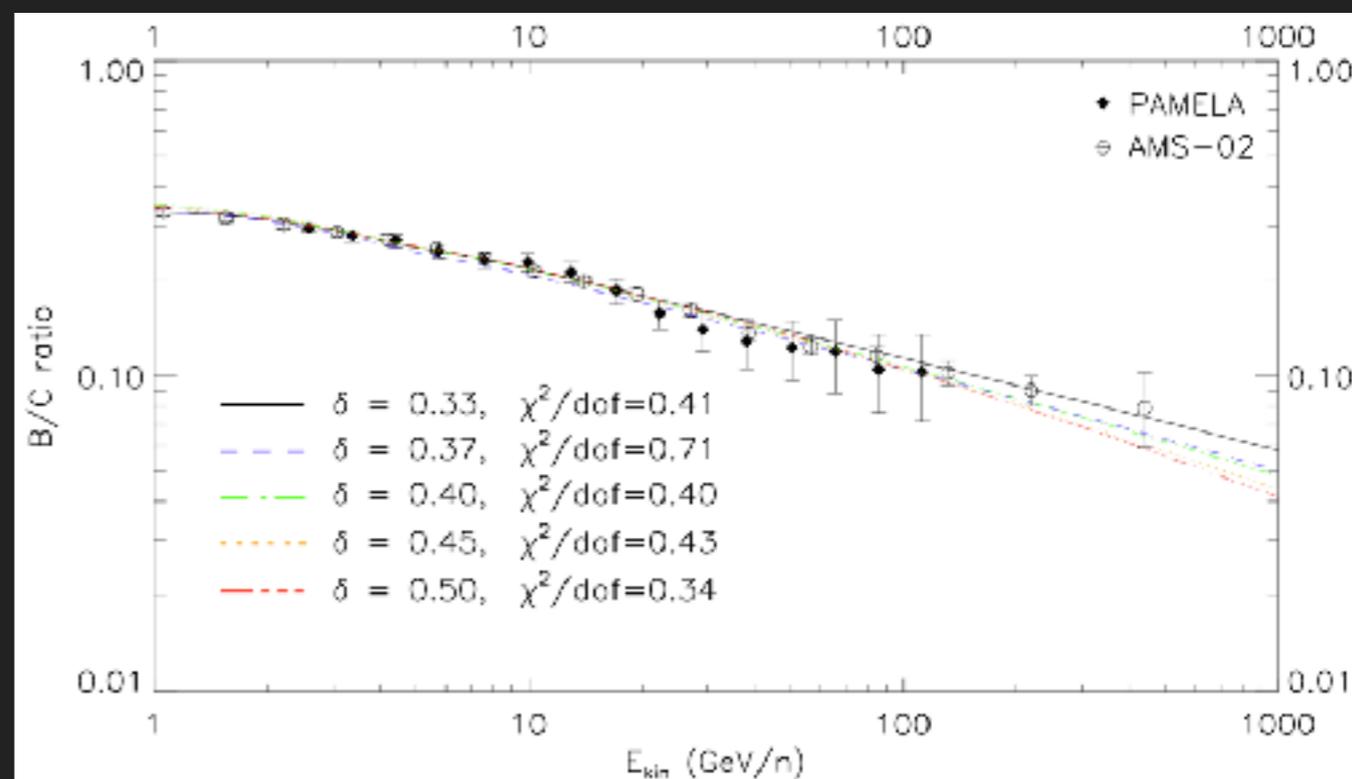
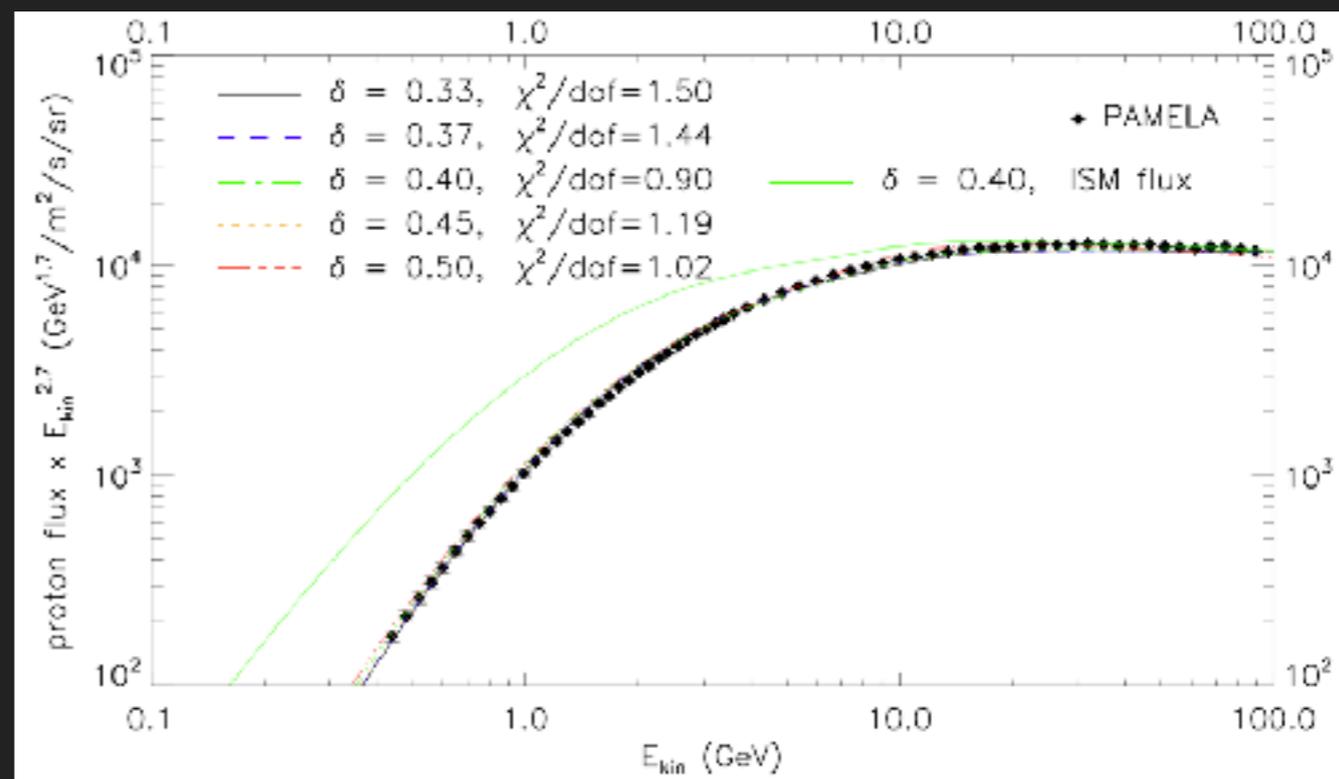
$$\phi_0 = 0.35 \text{ GV}, \phi_1 = 3.9 \text{ GV} \text{ and } R_0 = 0.5 \text{ GV}$$

$$\frac{dN^\oplus}{dE_{\text{kin}}}(E_{\text{kin}}) = \frac{(E_{\text{kin}} + m)^2 - m^2}{(E_{\text{kin}} + m + |Z| e\Phi)^2 - m^2} \frac{dN^{\text{ISM}}}{dE_{\text{kin}}}(E_{\text{kin}} + |Z| e\Phi)$$



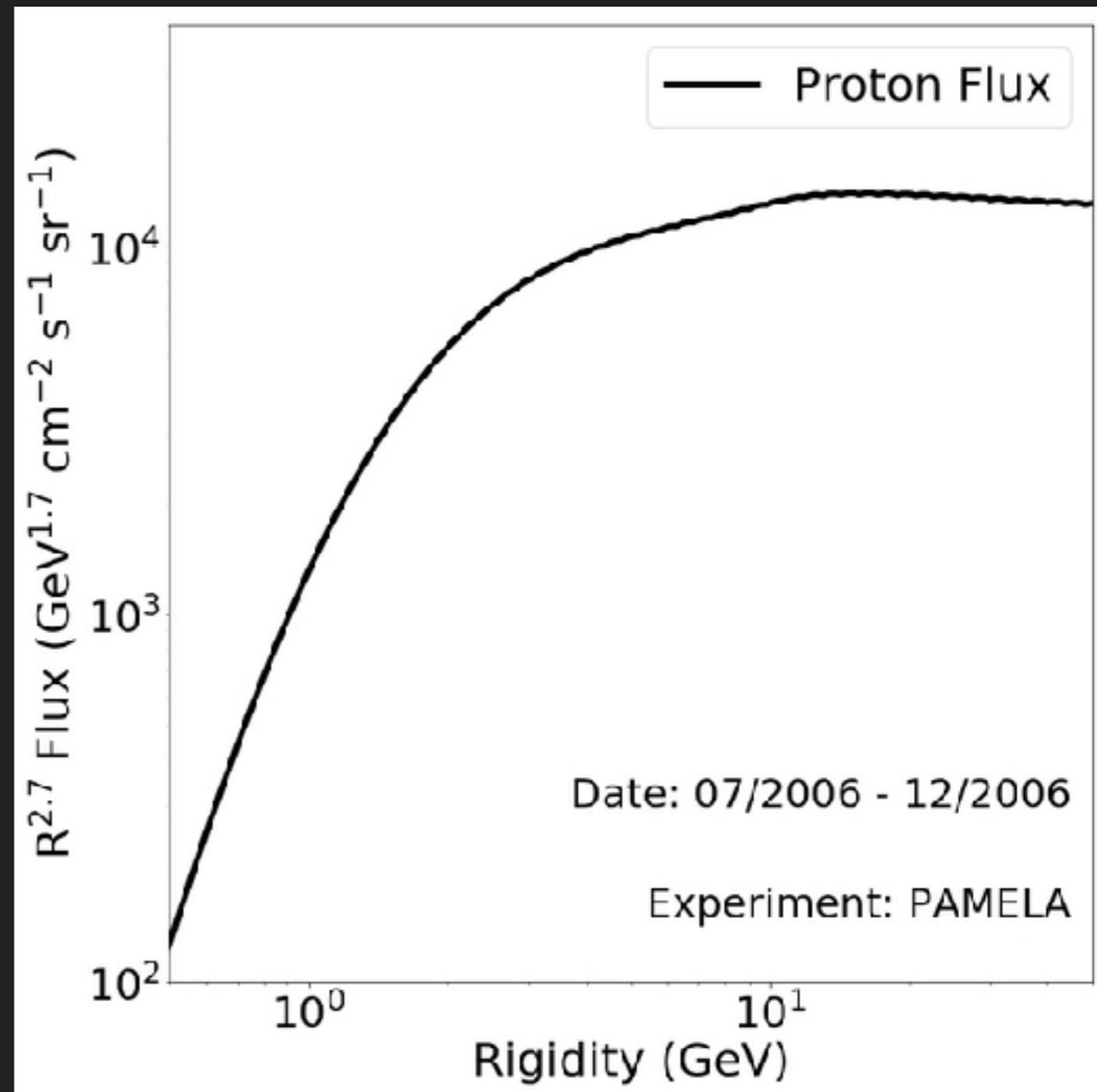
RESULTS: STUDYING THE INTERSTELLAR MEDIUM

- ▶ Using these models we can fit:
 - ▶ The proton spectrum from PAMELA
 - ▶ The B/C ratio from PAMELA and AMS-02
- ▶ Our models provide fits at the $\chi^2 / \text{d.o.f} \sim 1$ level.
- ▶ While a more complex theoretical model could be produced, it will be difficult to motivate with the data.

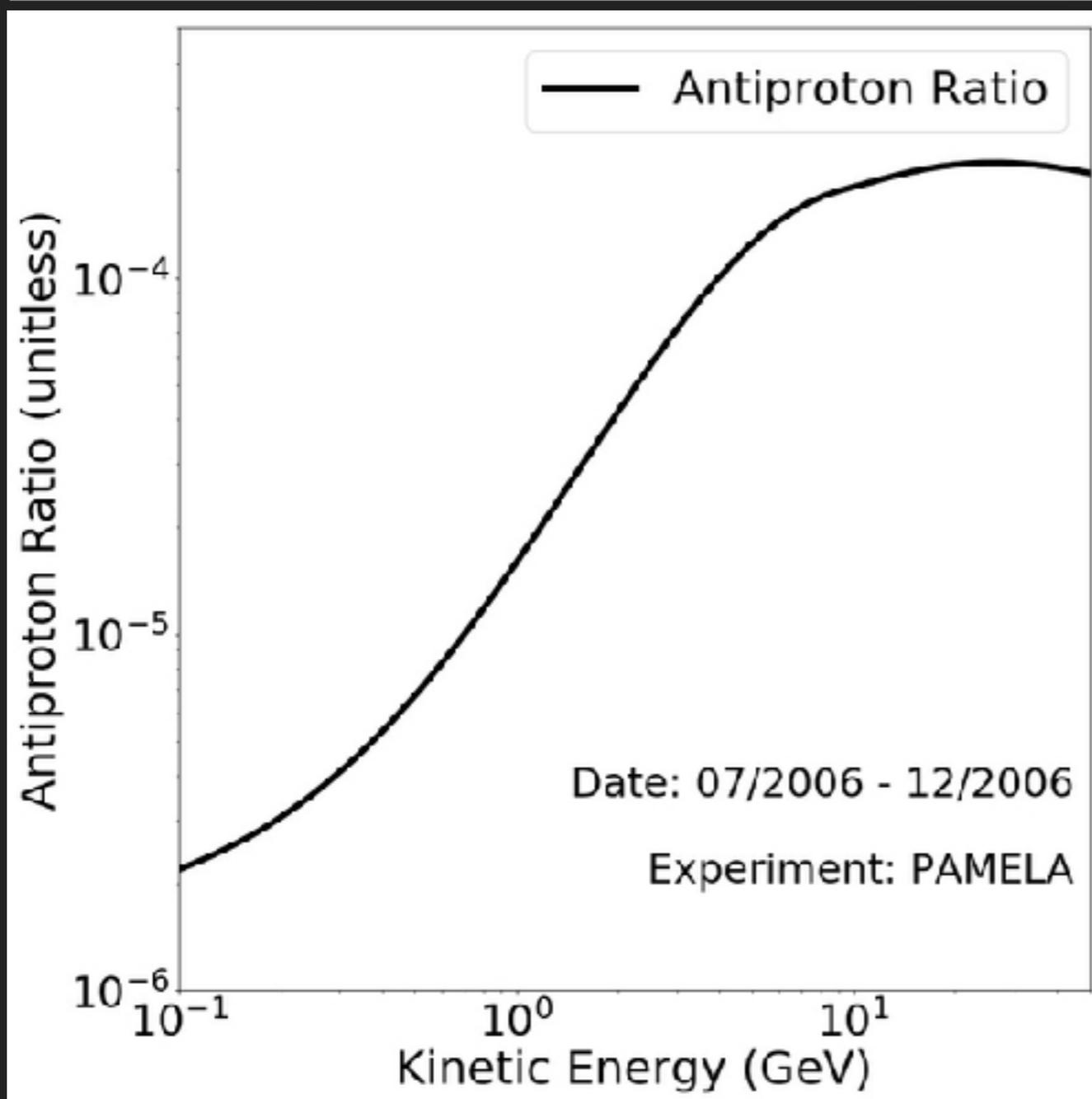


RESULTS: STUDYING THE INTERSTELLAR MEDIUM

- ▶ We can create time-dependent models for the observed proton flux.
- ▶ Has been fit with previous PAMELA data, could be compared with existing AMS-02 data.
- ▶ Model could be significantly refined through these comparisons.



THE TIME DEPENDENCE IN THE ANTIPROTON RATIO

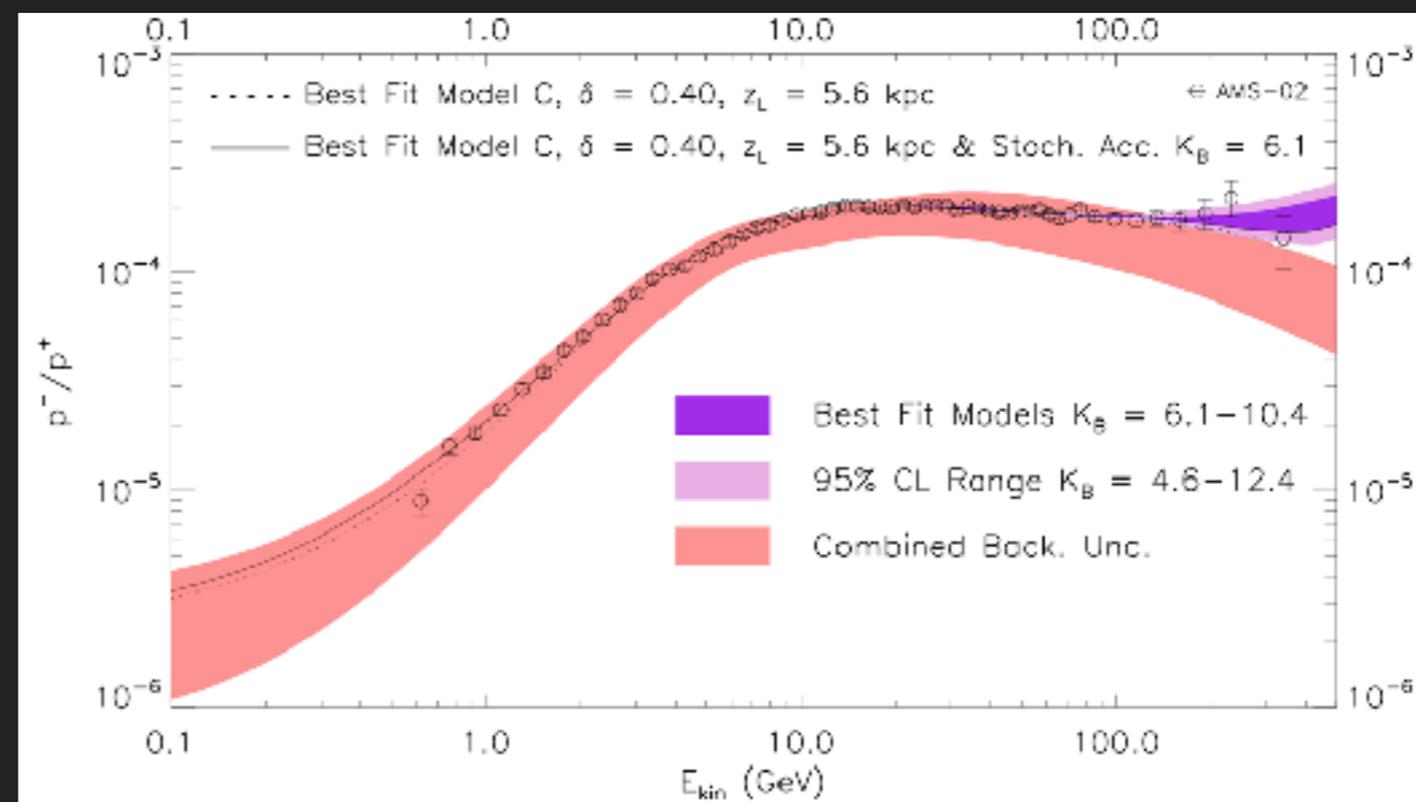
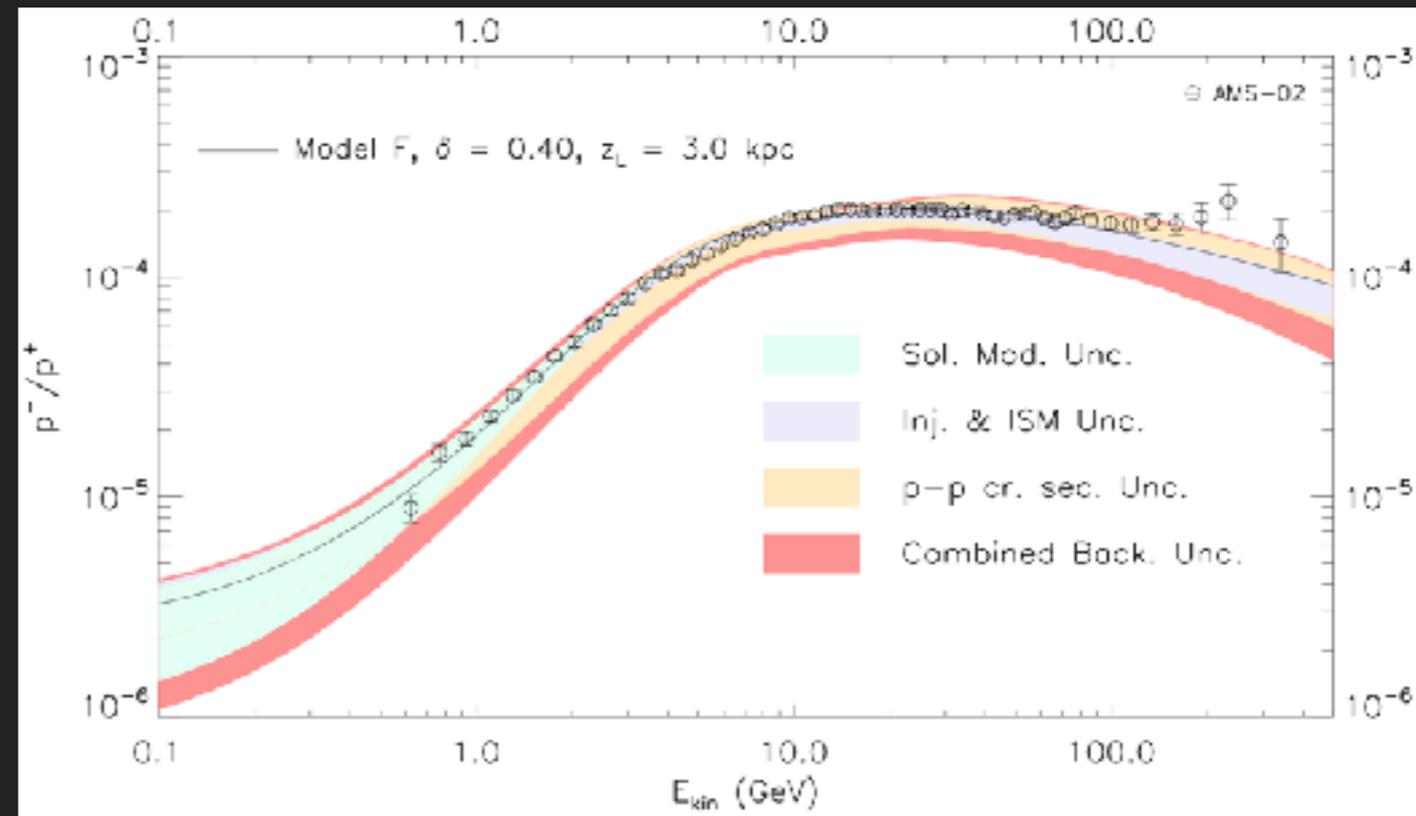


- ▶ Predictions for the antiproton ratio observed by PAMELA and AMS-02 as a function of time.
- ▶ Significant time-variability is observed, this significantly exceeds the uncertainty from current measurements.

- ▶ The time variability of all measured particle fluxes and ratios can be directly predicted in a similar fashion.

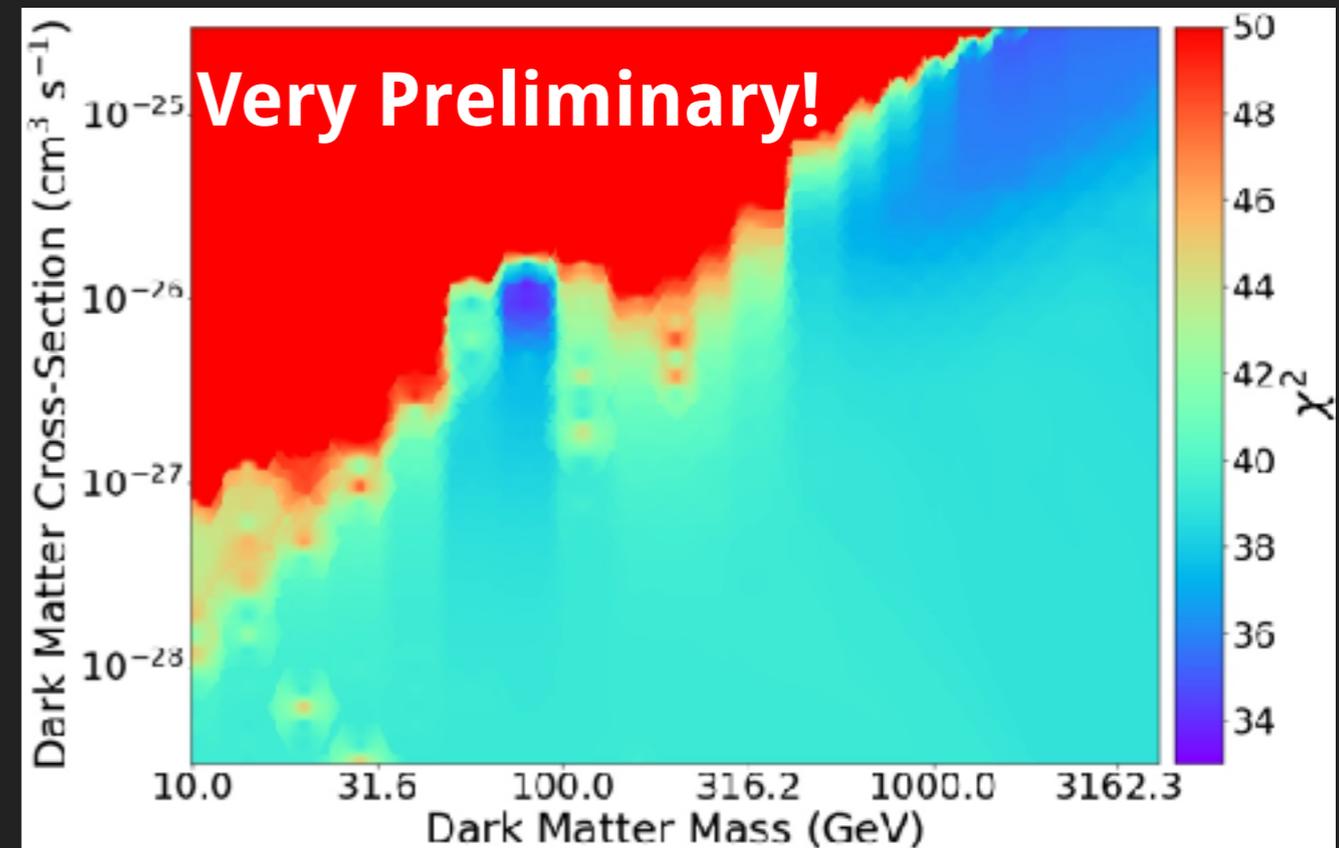
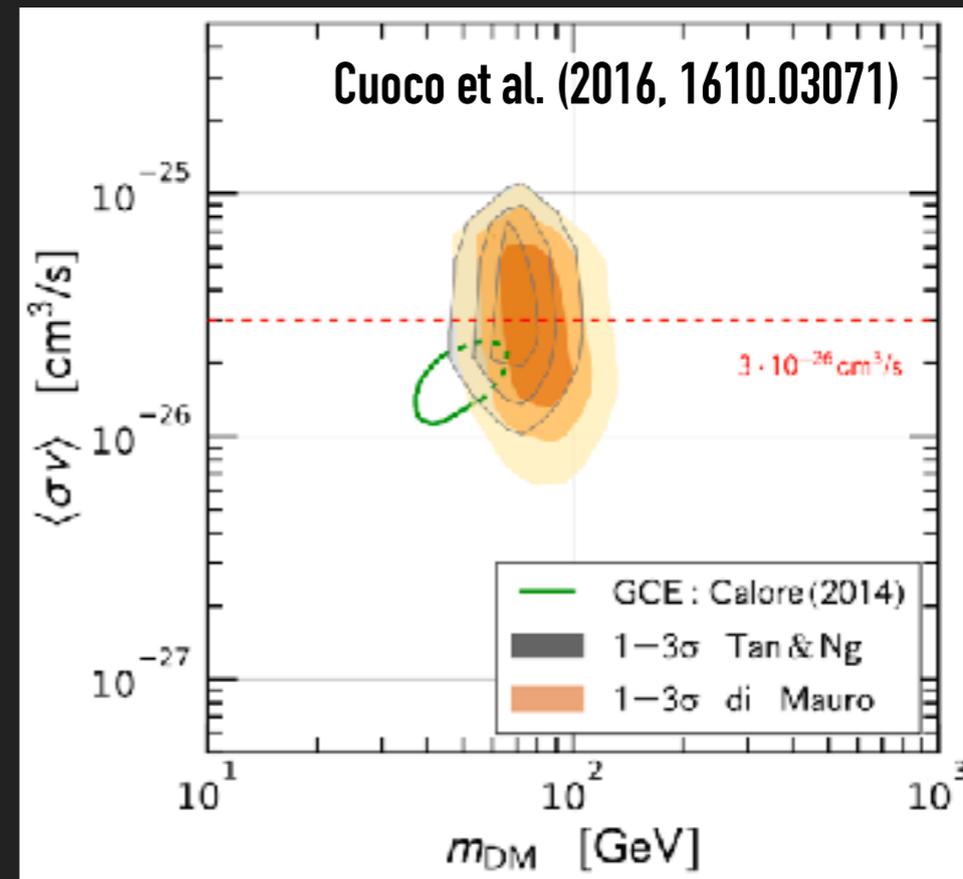
IMPLICATIONS: STOCHASTIC ACCELERATION OF COSMIC-RAYS

- ▶ Using these models (with no remaining degrees of freedom), we can fit:
 - ▶ The proton spectrum from PAMELA
 - ▶ The B/C ratio from PAMELA and AMS-02.
- ▶ The extremely precision of AMS-02 and PAMELA data make the accurate fit of low-energy CRs necessary to model high-energy behavior.



IMPLICATIONS: CONSTRAINTS ON DARK MATTER ANNIHILATION

- ▶ Using these models, we are also confirming previous claims of an antiproton excess at energies ~ 10 GeV.
- ▶ This is well fit by models of 80 GeV dark matter.
- ▶ More work remains to be done.



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TeVPA 2017
August 7 - 11



THE OHIO STATE UNIVERSITY
CENTER FOR COSMOLOGY AND
ASTROPARTICLE PHYSICS



Propose a Mini-Workshop!

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[🚀 Pre-Conference Mini-Workshops](#)

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Pre-Conference Mini-Workshops

We want to make TeVPA an opportunity for the community to get together to tackle open problems that require the combined input from different experimental collaborations and theorists.

To help achieve this, we are planning to host a number of informal pre-conference mini-workshop sessions, either on Saturday, August 5th or Sunday, August 6th. Each session would address a particular open problem. Potential topics are, for instance, "the anisotropic sky", "the Galactic Center excess", "high-energy astrophysical neutrino sources", and "UHECR sources"; the list is non-exhaustive. There would maybe be one or two short presentations. Most of the time should be dedicated to discussion and to collaboration within and between different experiments.

Attendees would be a subset of the TeVPA participants that are working on these problems or interested in them. Each session would be made up of members of cosmic-ray, gamma-ray, gravitational-wave, and neutrino collaborations, plus independent theorists. CCAPP would provide meeting rooms, facilities, and coffee breaks.

If you are interested in proposing, attending, or planning a mini-workshop broadly centered on TeV Particle Astrophysics, please contact us at tevpa2017@osu.edu.



CONCLUSIONS

- ▶ **We build a simple model that translates insights gained from computational models of solar modulation into an analytic form.**
- ▶ **This allows for the rapid computation of solar modulation, with results that are predictive and have few degrees of freedom.**
- ▶ **Updated observations from AMS-02, alongside upcoming solar data, will further refine these models.**
- ▶ **These models have already allowed for improved modeling of cosmic-ray propagation in the interstellar medium.**

EXTRA SLIDES

- ▶ **Extra Slides**

BREAKING THE DEGENERACY: PHYSICAL INTUITION

- ▶ We start with the diffusion equation, and consider particle propagation along and perpendicular to the heliospheric current sheet separately. We assume J_{source} is negligible at these energies.

$$\frac{\partial f}{\partial t} = -(\vec{V} + \langle \vec{v}_D \rangle) \nabla f + \nabla(\hat{D} \nabla f) + \frac{1}{3}(\nabla \vec{V}) \frac{\partial f}{\partial \ln p} + J_{\text{source}}$$

$$\lambda_d = r_{\text{Larmor}} \frac{(R/R_0)^2}{1 + (R/R_0)^2}$$

- ▶ Since the Larmor radius is inversely proportional to B, the propagation time (and total adiabatic energy loss) can be expressed as:

$$\tau_D \propto \frac{1}{|\langle \vec{v}_D \rangle|} \propto B(t) \frac{1 + (R/R_0)^2}{\beta (R/R_0)^3}$$